# Multi-Trait Matching and Gender Differentials in Intergenerational Mobility* 

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#### Abstract

We describe a model of multi-trait matching and inheritance in which individuals' attractiveness in the marriage market depends on their market and non-market characteristics. Gender differences in social mobility can arise if market characteristics are relatively more important in determining marriage outcomes for men than they are for women, and if they are more persistent across generations than non-market characteristics. A reduction in gender based discrimination in the labor market increases homogamy in the marriage market and lowers social mobility for both genders.


KEY WORDS: Social Mobility, Matching, Inheritance, Wage Gap
JEL Classification: C78, J12, J31, J62

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## 1 Introduction

We provide a theoretical rationale for the observed gender differential in intergenerational social mobility. ${ }^{1}$ We develop a simple model of two-sided matching and inheritance, in which individuals' attractiveness in the marriage market depends on their market and non-market traits. Market traits encompass individuals' characteristics that affect their earning potential in the labor market. Non-market traits encompass a range of other attributes that directly affect an individual's productivity in household production activities.

Market traits are, by definition, comparatively more dependent on the economic environment for their transmission than non-market traits are; ceteris paribus, this should make them intergenerationally more stable - for example, capital market imperfections that constrain human capital investment for the children of lower income individuals would imply that differences in cognitive ability between parents and their children would not readily translate into differences in earning ability. At the same time, persistent gender discrimination in the labor market implies that, ceteris paribus, market traits have a lower weight for women than they do for men in determining an individual's success in the marriage market.

In our model, the combination of these two asymmetries implies that women should be more socially mobile than men, and that a reduction in gender based inequality should lower intergenerational mobility for both men and women.

## 2 Multi-trait matching and inheritance

Consider a population of two genders, males and females, with an equal number of individuals of each gender who can only match with one individual of the opposite gender. Each individual possesses certain levels of two characteristics, $x$ and $y$. In our analysis, we think of $y$ as capturing various market-related traits, which directly affect an individual productivity in labor-market activities and thus his or her earning potential. The variable $x$ captures instead a range of other attributes that determine an individual's productivity in household production activities, but have little impact on labor market productivity.

[^1]Matching is gendered and involves nontransferabilities (Legros and Newman, 2007). For each individual, the levels of $x$ and $y$ combine to determine his or her attractiveness as a partner. In particular, the "desirability" of individual $i$ of gender $G=F, M$ and with characteristics $\left(x_{i}, y_{i}\right)$ is captured by the function $h_{i}^{G}\left(x_{i}, y_{i}\right), G=F, M$. This index provides an objective ranking for each individual of each gender in terms of his or her attractiveness to the other gender. Notice that the attractiveness function $h$ is genderspecific, since various factors can lead to differences in the relative importance of market and non-market characteristics for men and women. These factors include differential earnings in the labor market - which our analysis will focus on - but also biological differences in reproductive roles and the persistence of traditional gender roles within households.

We assume that non-market services (household production activities) can be substituted for by market services - but not the reverse. Suppose that $x$ represents non-market productivity expressed in money equivalent units (i.e., in terms of the cost of the substitute market services) and $y$ the unadjusted market productivity. Male and female market earning rates are denoted by $w^{M}$ and $w^{F}$, respectively. An individual's attractiveness, which depends on his or her contribution to a partnership, is then given by

$$
\begin{equation*}
h_{i}^{G}=x_{i}+w^{G} y_{i}, \quad G=F, M . \tag{1}
\end{equation*}
$$

Given a population of $n$ males and $n$ females, a matching equilibrium will feature (perfectly) positive assortative matching (Becker, 1973) in terms of gender-specific rank positions: the male with the highest $h^{M}$ will match with the female with the highest $h^{F}$, the male with the second highest $h^{M}$ will match with the female with the second highest $h^{F}$, and so on.

The inheritance process is modeled as follows. Each couple has two children, a daughter and a son. Inheritance of the two traits is assumed to be stochastic and to be captured by exogenous transition probabilities. These are the same across genders, but can differ across characteristics, reflecting both biological and institutional factors.

For simplicity, suppose that the process of inheritance is gender-segregated in the sense that daughters only inherit characteristics from their mothers and sons from their fathers. The level of non-market trait for a son (daughter) whose father (mother) has a level of a trait $c=x, y$ equal to $c^{\prime}$ is then

$$
\begin{equation*}
c^{\prime \prime}=c^{\prime}+\epsilon_{c} \tag{2}
\end{equation*}
$$

where $\epsilon_{c}(c=x, y)$ are independently distributed shock terms with values $\{-\delta, 0, \delta\}$ $(\delta>0)$. Denoting with $\bar{c}$ the mean level of a given trait, the probability of a positive shock $\left(\epsilon_{c}=\delta, c=x, y\right)$ is

$$
\pi_{c}=\left\{\begin{array}{cll}
\bar{\pi}_{c} & \text { if } & c^{\prime} \leq \bar{c}  \tag{3}\\
\underline{\pi}_{c}=\beta \bar{\pi}_{c} & \text { if } & c^{\prime}>\bar{c}
\end{array}\right.
$$

with $0<\beta<1$, implying $\underline{\pi}_{c}<\bar{\pi}_{c}$; the reverse being the case for negative shocks, i.e., the probability of a negative shock $\left(\epsilon_{c}=-\delta\right)$ is

$$
\pi_{c}=\left\{\begin{array}{cl}
\bar{\pi}_{c} & \text { if } \quad c^{\prime} \geq \bar{c}  \tag{4}\\
\underline{\pi}_{c}=\beta \bar{\pi}_{c} & \text { if } \quad c^{\prime}<\bar{c}
\end{array}\right.
$$

We assume that $\bar{\pi}_{c}+\underline{\pi}_{c}<1$, which guarantees that the stochastic process defined by (2) is stationary. ${ }^{2}$

The above formulation assumes that the shocks $\epsilon_{x}$ and $\epsilon_{y}$ are uncorrelated. This implies that the traits $x$ and $y$ will be independently distributed in the population in the long-run. If $n$ is large, the distribution of traits (and desirability levels) in the population will thus be invariant through time.

[^2]\[

$$
\begin{equation*}
z^{\prime \prime}=q y^{\prime \prime}+(1-q) z^{\prime} . \tag{5}
\end{equation*}
$$

\]

After integrating, this gives

$$
\begin{equation*}
z_{t}=\sum_{j=1}^{\infty}(1-q)^{j-1} q \sum_{i=-\infty}^{t-j+1} \epsilon_{i} \tag{6}
\end{equation*}
$$

a process that exhibits less time variability than the underlying process $y_{t}$ in our model.

## 3 Gender and social mobility

We focus on a scenario in which each trait can take one of two levels, high $(\bar{\gamma})$ and low $(\underline{\gamma})$, with $\underline{x}=\underline{y}=\underline{\gamma}, \bar{x}=\bar{y}=\bar{\gamma}, \delta=\bar{\gamma}-\underline{\gamma}, \bar{\pi}_{x}<1 / 2, \bar{\pi}_{y}<1 / 2$, and $\beta=0$.

Our analysis rests on two assumptions related to asymmetries between market and non-market traits. The first assumption has to do with the relative importance of these traits for men and women:

Assumption $1 w_{y}^{M}=\eta, \quad w_{y}^{F}=1 / \eta$, with $\eta>1$.
This implies that the $x$ trait has a higher weight in determining women's desirability than the $y$ trait does, with the reverse being the case for men. As mentioned in the introduction, recent studies show that non-market characteristic are indeed comparatively more important for women's attractiveness in the matching market than they are for men. In our model, the asymmetry derives from gender based discrimination in the labor market: lower earnings for females imply that their market skills are not as valuable in a partnership.

The second assumption has to do with an asymmetry in the degree of inheritability of market and non-market traits:

## Assumption $2 \quad \bar{\pi}_{x}>\bar{\pi}_{y}$.

This implies that the probability of transition from one level to the other is higher for the $x$ trait than for the $y$ trait - in a gender-neutral fashion.

As we show below, taken together Assumptions 1 and 2 result in the prediction that women are intergenerationally more mobile than men in terms of mating rank - and hence household income.

Given Assumption 1, the ranking in terms of attractiveness ( $h$ ) for individuals of different types will be different for males and females. Females and males of type $(\bar{x}, \bar{y})$ will be in the top (first) position and females and males of type ( $\underline{x}, \underline{y}$ ) in the bottom (fourth) position. However, for the second and third position, the ranking of types will be reversed for men and women: $(\bar{x}, \underline{y})$ type females and $(\underline{x}, \bar{y})$ type males will occupy the second position, while $(\underline{x}, \bar{y})$ type females and $(\bar{x}, \underline{y})$ type males will occupy the third position.

The long-run distribution of traits in a large population of $n$ individuals will then be as follows: $n / 2$ of all individuals will possess the high level of each of the two traits

Table 1: Matching with two traits

| Ranking of couples $(r)$ | Females | Males |
| :---: | :---: | :---: |
| 1 | $\bar{x}, \bar{y}$ | $\bar{x}, \bar{y}$ |
| 2 | $\bar{x}, \underline{y}$ | $\underline{x}, \bar{y}$ |
| 3 | $\underline{x}, \bar{y}$ | $\bar{x}, \underline{y}$ |
| 4 | $\underline{x}, \underline{y}$ | $\underline{x}, \underline{y}$ |

and $n / 2$ will possess the corresponding low level; moreover, as shocks are uncorrelated across the two traits, the number of individuals for each of the four possible combinations of trait levels will be $n / 4$. Assortative matching will give rise to the following ranking of couples ( $r$ ): 1) $(\bar{x}, \bar{y})$ females will be matched with $(\bar{x}, \bar{y})$ males; 2 ) $(\bar{x}, \underline{y})$ type females will be matched with $(\underline{x}, \bar{y})$ males; 3) $(\underline{x}, \bar{y})$ type females will be matched with $(\bar{x}, \underline{y})$ type males; 4) $(\underline{x}, \underline{y})$ type females will be matched with $(\underline{x}, \underline{y})$ type males. Couples occupying different ranks belong to different "social classes". Note that, if we take the comparatively lower weight on the market trait for women as implying comparatively lower earnings for women, then the ranking in terms of household income will be the same as the social ranking, i.e., couples belonging to a higher social class will have a higher level of income than couples in a lower class.

In this setting, "mixing", i.e., matching between individuals with different traits, arises only in the two middle social classes.

Consider a couple in the first position in the social (and income) ranking. The male offspring of this couple may remain in the same position or move to a lower social class: (a) with probability $\left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right)$, the son has traits $(\bar{x}, \bar{y})$ and remains in the first social class; (b) with probability $\bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right)$, the son has traits $(\underline{x}, \bar{y})$ and belongs to the second social class; (c) with probability $\left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y}$, the son has traits $(\bar{x}, \underline{y})$ and belongs to the third social class; (d) with probability $\bar{\pi}_{x} \bar{\pi}_{y}$, the son has traits $(\underline{x}, \underline{y})$ and belongs to the fourth social class.

Similarly, we can look at the chances of the female offspring of the same couple: (a) with probability $\left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right)$, the daughter has traits $(\bar{x}, \bar{y})$ and remains in the first social class; (b) with probability $\bar{\pi}_{y}\left(1-\bar{\pi}_{x}\right)$, the daughter has traits $(\bar{x}, \underline{y})$ and belongs to the second social class; (c) with probability $\bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right)$, the daughter has traits $(\underline{x}, \bar{y})$ and
belongs to the third social class; (d) with probability $\bar{\pi}_{x} \bar{\pi}_{y}$, the daughter has traits $(\underline{x}, \underline{y})$ and belongs to the fourth social class.

Proceeding in the same way for all cases, we obtain the following gender-specific transition probabilities, $\pi^{G}\left[r^{\prime}, r^{\prime \prime}\right]$, where $r^{\prime}$ represents the income ranking of the parents and $r^{\prime \prime}$ the income ranking of their offspring, and $G \in\{F, M\}$ :

$$
\begin{align*}
& \pi^{F}\left[r^{\prime}, r^{\prime \prime}\right]=\left[\begin{array}{cccc}
\left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x} \bar{\pi}_{y} \\
\left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x} \bar{\pi}_{y} & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) \\
\bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x} \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} \\
\bar{\pi}_{x} \bar{\pi}_{y} & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right)
\end{array}\right] ;  \tag{7}\\
& \pi^{M}\left[r^{\prime}, r^{\prime \prime}\right]=\left[\begin{array}{cccc}
\left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \bar{\pi}_{x} \bar{\pi}_{y} \\
\bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x} \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} \\
\left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \bar{\pi}_{x} \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right) & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) \\
\bar{\pi}_{x} \bar{\pi}_{y} & \left(1-\bar{\pi}_{x}\right) \bar{\pi}_{y} & \bar{\pi}_{x}\left(1-\bar{\pi}_{y}\right) & \left(1-\bar{\pi}_{x}\right)\left(1-\bar{\pi}_{y}\right)
\end{array}\right] . \tag{8}
\end{align*}
$$

It is straightforward to verify that, if the market trait is intergenerationally more persistent than the non-market trait $\left(\bar{\pi}_{x}>\bar{\pi}_{y}\right)$, daughters will be more likely to jump up or down in the social ranking compared to their brothers. Given the discreteness of the model, the difference is only in the "intermediate" jumps. In the example considered above, the daughter of a couple belonging to the first social class is more likely to jump down by two rank positions as compared to her brother (while the probability of a jump to the lowest social class is the same).

The mean correlation between the household income rank of a couple and that of an offspring of gender $G$ is obtained as

$$
\begin{equation*}
\frac{\operatorname{Cov}^{G}\left(r^{\prime}, r^{\prime \prime}\right)}{\sigma^{2}(r)}=\frac{\sum_{r^{\prime}} \sum_{r^{\prime \prime}} \pi^{G}\left[r^{\prime}, r^{\prime \prime}\right] r^{\prime} r^{\prime \prime} / 4-\mu(r)^{2}}{\sum_{r} r^{2} / 4-\mu(r)^{2}}, \quad G=F, M \tag{9}
\end{equation*}
$$

where $\operatorname{Cov}^{G}\left(r^{\prime}, r^{\prime \prime}\right)$ denotes the covariance between the matching rank of a parent and that of her offspring for gender $G, \sigma^{2}(r)$ is the variance of the rank, and $\mu(r)$ is mean rank.

The mechanism described above generates gender differences in intergenerational social mobility via the matching process, even if the inheritance process itself is the same for both genders: women are more likely to "marry up" (and "down") compared to men.

In the model, individuals are paired into couples that belong to different social classes,
depending on their market and non-market traits. Some of these individual characteristics are unobservable in the data. Nevertheless, as noted above, the model predicts a positive correlation between traits within the population, which implies that, even when focusing on observables - i.e., household income rather than social classes - women will be observed to be more mobile. To compare intergenerational income correlations for males and females, we can then examine the difference $\operatorname{Cov}^{F}\left(r^{\prime}, r^{\prime \prime}\right) / \sigma^{2}(r)-\operatorname{Cov}^{M}\left(r^{\prime}, r^{\prime \prime}\right) / \sigma^{2}(r)$, obtaining the following result:

Result 1 Under Assumptions 1 and 2, intergenerational mobility in household income is greater for females than it is for males.

Proof: The sign of the difference depends on the sign of the expression $\sum_{r^{\prime}} \sum_{r^{\prime \prime}}\left(\pi^{F}\left[r^{\prime}, r^{\prime \prime}\right]-\right.$ $\left.\pi^{M}\left[r^{\prime}, r^{\prime \prime}\right]\right) r^{\prime} r^{\prime}$. After simplification, we obtain

$$
\begin{equation*}
\sum_{r^{\prime}} \sum_{r^{\prime \prime}}\left(\pi^{F}\left[r^{\prime}, r^{\prime \prime}\right]-\pi^{M}\left[r^{\prime}, r^{\prime \prime}\right]\right) r^{\prime} r^{\prime \prime}=6\left(\bar{\pi}_{y}-\bar{\pi}_{x}\right) \tag{10}
\end{equation*}
$$

If $\bar{\pi}_{x}>\bar{\pi}_{y}$ (Assumption 2), then this expression is negative, implying a higher degree of intergenerational income mobility for females than for males.

This result provides a theoretical rationale for the aforementioned observed gender differential in social mobility.

An immediate implication of Result 1 is that a decrease in the degree of persistence of $y$ - as may result from institutional changes that promote earnings mobility (e.g., reforms aimed at alleviating credit constraints) - increases mobility for men more than it does for women:

Result 2 A switch from a scenario where Assumptions 1 and 2 are both satisfied to one where $\bar{\pi}_{y}^{\prime}=\bar{\pi}_{x}$-holding $\bar{\pi}_{x}$ constant - raises intergenerational income mobility for men more than it does for women.

Proof: The mapping between household matching rank, $r$, and household income is

$$
m(1)=2 \widetilde{w} \bar{\gamma}, \quad m(2)=w_{y}^{M} \bar{\gamma}+w_{y}^{F} \underline{\gamma}, \quad m(3)=w_{y}^{M} \underline{\gamma}+w_{y}^{F} \bar{\gamma}, \quad m(4)=2 \widetilde{w} \underline{\gamma} .
$$

Expression (9), after replacing the rankings $r^{\prime}$ and $r^{\prime \prime}$ with actual household market income levels, $m\left(r^{\prime}\right)$ and $m\left(r^{\prime \prime}\right)$, can be used to express intergenerational correlations with respect to income:

$$
\begin{equation*}
\frac{\operatorname{Cov}^{G}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=\frac{\sum_{r^{\prime}} \sum_{r^{\prime \prime}} \pi^{G}\left[r^{\prime}, r^{\prime \prime}\right] m\left(r^{\prime}\right) m\left(r^{\prime \prime}\right) / 4-\mu(m)^{2}}{\sum_{r} m(r)^{2} / 4-\mu(m)^{2}}, \quad G=F, M . \tag{11}
\end{equation*}
$$

For $\bar{\pi}_{y}^{\prime}=\bar{\pi}_{x}$, the corresponding expressions are the same for both genders and equal to

$$
\begin{equation*}
\frac{\operatorname{Cov}^{E}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=\frac{\sum_{r^{\prime}} \sum_{r^{\prime \prime}} \pi^{E}\left[r^{\prime}, r^{\prime \prime}\right] m\left(r^{\prime}\right) m\left(r^{\prime \prime}\right) / 4-\mu(m)^{2}}{\sum_{r} m(r)^{2} / 4-\mu(m)^{2}} \tag{12}
\end{equation*}
$$

where

$$
\pi^{E}\left[r^{\prime}, r^{\prime \prime}\right]=\left[\begin{array}{cccc}
\left(1-\bar{\pi}_{x}\right)^{2} & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \left(\bar{\pi}_{x}\right)^{2}  \tag{13}\\
\bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \left(1-\bar{\pi}_{x}\right)^{2} & \left(\bar{\pi}_{x}\right)^{2} & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) \\
\bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \left(\bar{\pi}_{x}\right)^{2} & \left(1-\bar{\pi}_{x}\right)^{2} & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) \\
\left(\bar{\pi}_{x}\right)^{2} & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \bar{\pi}_{x}\left(1-\bar{\pi}_{x}\right) & \left(1-\bar{\pi}_{x}\right)^{2}
\end{array}\right] .
$$

The differences in correlations between the two scenarios for each gender can then be written (after simplification) as

$$
\begin{align*}
& \frac{\operatorname{Cov}^{E}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}-\frac{\operatorname{Cov}^{F}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=-\left(\bar{\pi}_{x}-\bar{\pi}_{y}\right) \frac{2\left(w_{y}^{F}\right)^{2}}{\left(w_{y}^{F}\right)^{2}+\left(w_{y}^{M}\right)^{2}}<0  \tag{14}\\
& \frac{\operatorname{Cov}^{E}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}-\frac{\operatorname{Cov}^{M}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=-\left(\bar{\pi}_{x}-\bar{\pi}_{y}\right) \frac{2\left(w_{y}^{M}\right)^{2}}{\left(w_{y}^{F}\right)^{2}+\left(w_{y}^{M}\right)^{2}}<0 . \tag{15}
\end{align*}
$$

For $w_{y}^{M}>w_{y}^{F}$ (Assumption 1), the change for men is greater (in absolute value) than the corresponding change for women.

A second implication is that a reduction in the gender earnings gap, $w_{y}^{M}-w_{y}^{F}$, will reduce intergenerational mobility for both men and women. To see this, compare a scenario where $w_{y}^{M}>1>w_{y}^{F}$, and $\left(w_{y}^{M}+w_{y}^{F}\right) / 2=\widetilde{w}$ - giving rise to "mixing" of types in the middle two social positions - with one where mean earnings are also $\widetilde{w}$ but where $w_{y}^{M}=w_{y}^{F}=\widetilde{w}$. In the latter scenario, if $\widetilde{w}>1$ the market trait always dominates the non-market trait, and so the ranking of $(\underline{x}, \bar{y})$ type individuals will be higher than that of $(\underline{x}, \bar{y})$ type individuals for both genders; if $\widetilde{w}<1$, the reverse will be true. In either case no "mixing" will occur:

Result 3 In the absence of gender discrimination in the labor market $\left(w_{y}^{M}=w_{y}^{F}\right)$, matching will lead to perfect homogamy.

If $\widetilde{w}>1$, since the market trait dominates the non-market trait, households in the second position of the social ranking always have a higher income than those in the third social position, irrespectively of whether or not a gender earnings gap is present and mixing occurs. However, the income gap between households in the second and third positions of the overall rank is smaller when a gender earnings gap is present and mixing occurs than under equal earning rates and perfect homogamy. Given that transition probabilities for males remain the same in both scenarios, this implies a smaller $\operatorname{Cov}^{G}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right) / \sigma^{2}(m)$ for males as well as for females in the latter scenario in comparison with the former. The same is true for $\widetilde{w}<1$ - in this case, households in the second position have a lower income than those in the third position, but since income rank is everywhere decreasing with social rank, the same conclusion applies - i.e., income mobility is reduced for both females and males.

Result 4 A switch from a scenario where Assumptions 1 and 2 are both satisfied to one where $w_{y}^{M}=w_{y}^{F}$ results in lower intergenerational income mobility for both genders.

Proof: Assume $\widetilde{w}>1$. The mapping between household matching rank, $r$, and household income under mixing and $w_{y}^{M}>w_{y}^{F}$, and the resulting mean intergenerational correlations in household income for women and men, are as derived in the proof of Result 2. For $w_{y}^{M}=w_{y}^{F}$ (and no mixing), the corresponding mapping between mating rank and household income is

$$
m_{N}(1)=m_{N}(2)=2 \widetilde{w} \bar{\gamma}, \quad m_{N}(3)=m_{N}(4)=2 \widetilde{w} \underline{\gamma} .
$$

Transition probabilities in this case are the same for both genders and equal to the transition probabilities $\pi^{M}\left[r^{\prime}, r^{\prime \prime}\right]$ that apply to males under mixing. The mean intergenerational correlation in household income (the same for both genders) is then

$$
\begin{equation*}
\frac{\operatorname{Cov}^{N}\left(m_{N}\left(r^{\prime}\right), m_{N}\left(r^{\prime \prime}\right)\right)}{\sigma^{2}\left(m_{N}\right)}=\frac{\sum_{r^{\prime}} \sum_{r^{\prime \prime}} \pi^{G}\left[r^{\prime}, r^{\prime \prime}\right] m_{N}\left(r^{\prime}\right) m_{N}\left(r^{\prime \prime}\right) / 4-\mu\left(m_{N}\right)^{2}}{\sum_{r} m_{N}(r)^{2} / 4-\mu\left(m_{M}\right)^{2}} . \tag{16}
\end{equation*}
$$

The differences in correlations between the two scenarios for each gender can then be written (after simplification) as

$$
\begin{equation*}
\frac{\operatorname{Cov}^{N}\left(m_{N}\left(r^{\prime}\right), m_{N}\left(r^{\prime \prime}\right)\right)}{\sigma^{2}\left(m_{N}\right)}-\frac{\operatorname{Cov}^{F}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=\left(\bar{\pi}_{x}-\bar{\pi}_{y}\right) \frac{2\left(w_{y}^{M}\right)^{2}}{\left(w_{y}^{F}\right)^{2}+\left(w_{y}^{M}\right)^{2}}>0 ; \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\operatorname{Cov}^{N}\left(m_{N}\left(r^{\prime}\right), m_{N}\left(r^{\prime \prime}\right)\right)}{\sigma^{2}\left(m_{N}\right)}-\frac{\operatorname{Cov}^{M}\left(m\left(r^{\prime}\right), m\left(r^{\prime \prime}\right)\right)}{\sigma^{2}(m)}=\left(\bar{\pi}_{x}-\bar{\pi}_{y}\right) \frac{2\left(w_{y}^{F}\right)^{2}}{\left(w_{y}^{F}\right)^{2}+\left(w_{y}^{M}\right)^{2}}>0 \tag{18}
\end{equation*}
$$

Proceeding in the same way for the case $\widetilde{w}<1$, we arrive at the same conclusion. So, a reduction in the earnings gap that induces homogamy lowers intergenerational income mobility for both genders.

These results suggests that labor market reforms that have led to a narrowing of gender wage gaps may be one of the reasons behind the increased educational homogamy observed in recent decades; and that, by tightening the link between the social and economic status of parents and their offspring, such policies could paradoxically make society more unequal.

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[^1]:    ${ }^{1}$ See, for example, Hirvonen (2008).

[^2]:    ${ }^{2}$ In the above specification the inheritance process is differentiated for the two traits, with the difference reflecting institutional factors that are left unmodeled. An analogous formulation would be one where inheritance is identical for the two traits, but where market productivity depends on intrinsic ability, as represented by the $x$ trait, as well as on educational attainment, which in turn can be limited by parental income (e.g., because of imperfect capital markets). For example, the matching attractiveness of an offspring with characteristics $\left(x^{\prime \prime}, y^{\prime \prime}\right)$ could be written as $h^{\prime \prime}=x^{\prime \prime}+w z^{\prime \prime}$, where $z^{\prime \prime}$ depends positively both on $y^{\prime \prime}$ and on parental income $w z^{\prime}$, according to the mapping $z^{\prime \prime}=q y^{\prime \prime}+(1-q) z^{\prime}$. Market productivity would then be "inherited" according to the following process:

